

Intrinsic Excitation of Even-Even Nuclei*

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Intrinsic excitations of the rotational levels of even-even nuclei and the intrinsic structure of the wave functions are investigated. The rotational levels of negative parity are treated on the same basis as the levels of positive parity. The experimental energy levels are analyzed in order to obtain information about the intrinsic excitation in excited states of even-even nuclei.

I. INTRODUCTION

THE complex spectra of nuclei have successfully been accounted for by the Bohr model,^{1,2} especially in the rotational region. In this model, the underlying assumption is that the nucleus consists of a core of particles, tightly coupled together, and an extra particle or group of particles rather loosely bound to this core. The core is generally deformed from the spherical shape, and its rotation gives rise to rotational levels.

Further progress in nuclear phenomenology has been achieved by Davidov and others.³⁻⁵ They have been able to account for the level schemes and transition probabilities in the vibrational region as well as in the rotational region. In the Davidov-Filippov model, the presence of extra particles is not assumed.⁶ The wave function of the nucleus is supposed to be invariant under a rotation of 180° around the coordinate axes fixed to the nucleus, so this model does not apply in the case of negative parity.

In the case of negative parity, the bands are presumed to be due to the presence of octupole vibration,⁷ with the result that the corresponding wave function transforms quite differently from that of the Davidov-Filippov model. It would be hoped, of course, that theory could treat any band on the same basis.

In the present article we intend to discuss the excitation of even-even nuclei from a systematic point of view in which all bands are treated on the same basis; differences between them are presumed to be due primarily to different excitation of the extra particles.

In Sec. II, the relation between the energy spectrum

and the intrinsic excitation is discussed. In Secs. III and IV experimental energy levels are analyzed on the basis of Sec. II. In Sec. V, some remarks on γ transitions are presented.

II. INTRINSIC EXCITATIONS AND ENERGY LEVELS

A. Model Adopted in the Present Paper

In nuclear physics, it is often assumed that a single particle or group of particles, such as an alpha particle, are rather loosely bound to the remaining core. We will adopt such a model, but with the understanding that the core is not necessarily a closed shell, and that the other particles are coupled intrinsically in the singlet state, and insofar as a few bands near the ground state are concerned, behave as a single particle. We shall assume that with respect to the body-fixed frame the third component Ω of the angular momentum of the extra particles is mainly zero,⁸ although in general, this quantity is not a good quantum number. The assumptions with respect to the core are that (1) the spin of the core is zero; (2) insofar as we neglect the effect of the Coriolis force, the wave function of the core is invariant under any rotation of 180° around the coordinate axes; and (3) the excitation of the core appears as the rotational energy. Thus, the present model is just the Bohr model^{1,2} although we shall not adopt the liquid drop picture of the core.

B. Classification of the Wave Function

When the angular wave function $\Psi_{IM}(y)$ of the total system expressed in the space-fixed frame of reference (the y frame) is projected onto the axes fixed to the core (the z frame), it is expressed as⁹

$$\begin{aligned} \Psi_{IM}(y) &= \sum_K \mathcal{D}_{MK}^{I*} \Psi_{IK}(z) \\ &= \sum_K \sum_L \sum_l \sum_\Omega \mathcal{D}_{MK}^{I*} (lL\Omega K - \Omega | lLlK) \\ &\quad \times \chi_{l\Omega} \Phi_{L, K-\Omega}. \end{aligned} \quad (2.1)$$

Here $\chi_{l\Omega}$ is the angular wave function of the extra

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¹ A. Bohr, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 26, No. 14 (1952).

² A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. 27, No. 16 (1953).

³ A. S. Davidov and G. F. Filippov, Nucl. Phys. 8, 237 (1958).

⁴ A. S. Davidov and V. S. Rostrovsky, Nucl. Phys. 12, 58 (1959).

⁵ A. S. Davidov, N. S. Robotnov, and A. A. Chaban, Nucl. Phys. 12, 169 (1960).

⁶ We are confining our discussion to even-even nuclei.

⁷ K. Adler, A. Bohr, T. Huus, B. Mottelson, and A. Winther, Rev. Mod. Phys. 28, 432 (1956).

⁸ Reference 2, p. 97.

⁹ The definition of the \mathcal{D}_{MK}^{I*} -function is identical to that by Rose [M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957)].

particles relative to the center mass of the core; $\Phi_{L,K-\Omega}$ is the angular wave function of the core.

In what follows, we shall add the superscript I , referring to the total angular momentum of the system, to $\Phi_{L,K-\Omega}$ (whence $\Phi_{L,K-\Omega}^I$), and we shall let $\Phi_{L,K-\Omega}^I$ include the radial wave function of both the extra particles and the core in addition to the angular part of the core. Furthermore, we shall assume that $\Phi_{L,K-\Omega}^I$ includes various mixing parameters with respect to L, l and Ω . At the same time, we shall understand that $\Psi_{IM}(y)$ is now the total wave function in the y frame.

TABLE I. Character of four types of the irreducible representation of the D_2 group. For notation see Eq. (2.10) in the text. Capital letters describe the type of core and lower case letters the type of extra particles.

Type		(E)	(C_2^x)	(C_2^y)	(C_2^z)
A	a	1	1	1	1
B_1	b_1	1	-1	-1	1
B_2	b_2	1	-1	1	-1
B_3	b_3	1	1	-1	-1

Introducing the wave function $\Phi_{L,K}^{I(\pm)}$ for $K \neq 0$, by

$$\Phi_{L,K}^{I(\pm)} = (1/\sqrt{2})(\Phi_{L,K}^I \pm \Phi_{L,-K}^I), \quad (2.2)$$

and the notation

$$\mathcal{D}_{MK}^{I(\pm)*} = (1/\sqrt{2})(\mathcal{D}_{MK}^{I*} \pm (-)^{l+L-I}\mathcal{D}_{M-K}^{I*}), \quad (2.3)$$

the wave function $\Psi_{IM}(y)$ separates as follows:

$$\Psi_{IM}(y) = \Psi_{IM}^{(1)} + \Psi_{IM}^{(2)} + \Psi_{IM}^{(3)}, \quad (2.4)$$

where

$$\Psi_{IM}^{(1)} = \sum_{LI} \mathcal{D}_{M0}^{I*}(lL00|lLI0)\chi_{l0}\Phi_{L0}^I,$$

and

$$\Psi_{IM}^{(2)} = \sum_{LI} \sum_{K>0} (lL0K|lLIK) \times \chi_{l0}(\mathcal{D}_{MK}^{I(+)*}\Phi_{LK}^{I(+)} + \mathcal{D}_{MK}^{I(-)*}\Phi_{LK}^{I(-)}).$$

The wave function $\Psi_{IM}^{(3)}$ contains all terms with $\Omega > 0$. We shall neglect the contributions from this wave function, assuming them to be small. (The wave function $\Psi_{IM}^{(3)}$ is described in the Appendix.)

We shall now examine the the rotational properties of the wave function. Let us consider the D_2 group,¹⁰ which consists of rotations of the order 2, $C_2^x, C_2^y,$ and C_2^z , about three axes fixed to the core. Let $U(xyz)$ be any wave function referred to the frame fixed to the core. Then C_2^x, C_2^y, C_2^z are defined by

$$\begin{aligned} C_2^x U(xyz) &= U(x-y-z), \\ C_2^y U(xyz) &= U(-xy-z), \end{aligned} \quad (2.5)$$

and

$$C_2^z U(xyz) = U(-x-yz),$$

¹⁰ L. D. Landau and E. M. Lifshitz, *Quantum Mechanics* (Addison-Wesley Publishing Company, Reading, Massachusetts, 1958).

or in terms of Euler angles,

$$\begin{aligned} C_2^x(\theta\varphi\psi) &= (\theta+\pi, \varphi, -(\psi+\pi)), \\ C_2^y(\theta\varphi\psi) &= (\theta-\pi, \varphi, -\psi), \end{aligned} \quad (2.6)$$

and

$$C_2^z(\theta\varphi\psi) = (\theta, \varphi, \psi+\pi).$$

These properties lead to the following relations:

$$\begin{aligned} C_2^x \mathcal{D}_{MK}^{I*} &= (-)^I \mathcal{D}_{M-K}^{I*}, \\ C_2^y \mathcal{D}_{MK}^{I*} &= (-)^{I+K} \mathcal{D}_{M-K}^{I*}, \\ C_2^z \mathcal{D}_{MK}^{I*} &= (-)^K \mathcal{D}_{MK}^{I*}, \end{aligned} \quad (2.7)$$

$$C_2^x \chi_{l0} = (-)^{-l} \chi_{l0}, \quad C_2^y \chi_{l0} = (-)^{-l} \chi_{l0}, \quad C_2^z \chi_{l0} = \chi_{l0}, \quad (2.8)$$

and

$$\begin{aligned} C_2^x \Phi_{LmL}^{I(\pm)} &= \pm (-)^{-L} \Phi_{LmL}^{I(\pm)}, \\ C_2^y \Phi_{LmL}^{I(\pm)} &= \pm (-)^{-(L+mL)} \Phi_{LmL}^{I(\pm)} \quad (\text{for } m_L \neq 0), \\ C_2^z \Phi_{LmL}^{I(\pm)} &= (-)^{-mL} \Phi_{LmL}^{I(\pm)}, \\ C_2^x \Phi_{L0}^I &= (-)^{-L} \Phi_{L0}^I, \quad (\text{for } m_L = 0), \\ C_2^y \Phi_{L0}^I &= (-)^{-L} \Phi_{L0}^I, \\ C_2^z \Phi_{L0}^I &= \Phi_{L0}^I. \end{aligned} \quad (2.9)$$

Note that the parity of a state is given by $(-)^{l+L}$.

TABLE II. Classification of the wave function $\Psi_{IM}^{(1)}(K=\Omega=0)$. In this case $I+l+L$ is even due to the property of the Clebsch-Gordan coefficient. The boldface characters show the wave functions chosen in the text. Thus, (a) the band with $K=0^+$; $I=0^+, 2^+, 4^+, \dots$ (l =even, L =even), and (b) the band with $K=0^-$; $I=1^-, 3^-, 5^-, \dots$ (l =odd, L =even) remains, if we consider that the wave function of the core belongs to the type A .

I	l	Parity	Type of wave function
even	even	+	Aa
even	odd	+	$B_1 b_1$
odd	even	-	$B_1 a$
odd	odd	-	Ab₁

Introducing the notation

$$\begin{aligned} (E) &= (U|1|U), \\ (C_2^x) &= (U|C_2^x|U), \\ (C_2^y) &= (U|C_2^y|U), \\ (C_2^z) &= (U|C_2^z|U), \end{aligned} \quad (2.10)$$

we classify the D_2 group into four irreducible representations according to the corresponding transformation property. This is shown in Table I. This classification is applied to $\Psi_{IM}^{(1)}$ and $\Psi_{IM}^{(2)}$, the results being shown in Tables II and III, respectively.

We observe that the wave function can be other than type A insofar as the invariance of the total wave function written in the y frame with respect to rotation within the z frame is concerned. This can be shown by using Eqs. (2.1), (2.7), (2.8), and (2.9), together with

TABLE III. Classification of the wave function $\Psi_{IM}^{(2)}(K>0, \Omega=0)$. The boldface characters show the wave function chosen in the text.

K	l	Parity	$L=\text{even}$		Parity	$L=\text{odd}$	
			$\Phi_{LK}^{I(+)}\chi_{l0}$	$\Phi_{LK}^{I(-)}\chi_{l0}$		$\Phi_{LK}^{I(+)}\chi_{l0}$	$\Phi_{LK}^{I(-)}\chi_{l0}$
even >0	even	+	Aa	B_1a	-	B_1a	Aa
	odd	-	Ab₁	B_1b_1	+	B_1b_1	Ab_1
odd	even	+	B_3a	B₂a	-	B₂a	B_3a
	odd	-	B_3b_1	B₂b₁	+	B₂b₁	B_3b_1

the following property of the Clebsch-Gordan coefficient :

$$(aba\beta|abc\gamma) = (-)^{a+b-c}(ab-\alpha-\beta|abc-\gamma). \quad (2.11)$$

We are free to choose any type for the core wave function. However, we choose type A since this has the most symmetric form.

Once we adopt the wave function of type A , the K value should be even for all energy levels as seen in Tables II and III. Table II shows that for $K=0^\pm$, L assumes only even values. From Table III, one sees that for the band $K=2^+$ either $L=\text{even}$, $l=\text{even}$ or $L=\text{odd}$, $l=\text{odd}$ or a mixture of both, and for the band $K=2^-$ either $L=\text{even}$, $l=\text{odd}$ or $L=\text{odd}$, $l=\text{even}$ or a mixture of both. The rotational wave function is given by $\mathfrak{D}_{MK}^{I(+)*}$ for $L=\text{even}$ and by $\mathfrak{D}_{MK}^{I(-)*}$ for $L=\text{odd}$.

To determine what kind of L is most probable for a given band, we shall consider the Schrödinger equation

$$(H_0 + H_{\text{rot}})\Psi = E\Psi. \quad (2.12)$$

The rotational Hamiltonian H_{rot} can be expressed by Euler angles, but the intrinsic Hamiltonian H_0 does not include such coordinates. Correspondingly, $\mathfrak{D}_{MK}^{I(\pm)*}$ is expressed in terms of Euler angles, while $\Phi_{MK}^{I(\pm)}$ is described only by intrinsic coordinates. Putting $\Psi_{IM}^{(2)}$ into (2.12), orthogonality of $\mathfrak{D}_{MK}^{I(+)*}$ to $\mathfrak{D}_{MK}^{I(-)*}$ immediately leads to the result that wave functions of different even-oddness with respect to L cannot be mixed, but, rather, belong to two separate bands.¹¹ Since in the bands $K=0^\pm$, L assumes only even values and since bands $K=0^-, 2^\pm$ appear very closely, as in W^{182} , for example, it is most natural to assume that L takes on only even values for $K=2^\pm$. (The tentative assignment¹² for the K value of the level sequence $1^-, 3^-, 5^-, \dots$ of W^{182} is 1^- . The possibility that this might be 0^- as is the case for Ra^{226} ,¹³ will be discussed in Sec. IV C.)

In conclusion, $L=\text{even}$ for all bands under consideration and $l=\text{even}$ (odd) for positive (negative) parity bands. It is interesting to note that l should not be zero for the band $K=2^+$, since otherwise we would have the energy spectrum $I=2^+, 4^+, 6^+, \dots$ in contradiction with experiment. Therefore, l is even and equal to or greater than 2 for $K=2^+$. On the other hand, l may be 0

for $K=0^+$. It is for this reason that we have considered the presence of extra particles besides the rotating core. In the Davidov-Filippov model, the intrinsic wave function of the band $K=2^+$ was assumed to be the same as that for $K=0^+$. This does not always seem to be justified.

From the above discussion, it is very tempting to assume that the bands $K=0^+, 0^-, 2^+, 2^-$ are characterized by $l=0, 1, 2$, and 3, respectively, in an extension of the work by Nilsson^{14,15} and Mottelson¹⁵ for the case of even-odd nuclei. For our present purposes, however, it will not be necessary to develop this idea any further, since the experimental analysis (Secs. III and IV) requires only a knowledge of the even-oddness of l .

In preparing for the following sections, we have to discuss general features of the rotational Hamiltonian H_{rot} in Eq. (2.12).

It can be expressed quite generally as

$$H_{\text{rot}} = \left(\frac{\hbar^2}{2}\right) \sum_{a=1}^3 \frac{L_a^2}{M_a^2}, \quad (2.13)$$

where L_a is the angular momentum operator of the system, which can be expressed by the Euler angles and M_a will be an operator expressed by the intrinsic coordinates; i.e., the coordinates which describe the residual degree of freedom and, thus, which describes the intrinsic motion in the body-fixed frame of reference. The orthogonality of the set of the Euler angles to the set of the intrinsic coordinates in the $3N$ -dimensional coordinate space leads to the following important properties of the operator M_a : Firstly, $[M_a, L_a]=0$, or $[M_a^{-1}, L_a]=0$. Secondly, M_a couples the states only of the same I . Finally, M_a may couple states of different K as well as the states of the same K , because different K means the intrinsic excitation. The first property of M_a enables us to write the rotational Hamiltonian H_{rot} simply as

$$H_{\text{rot}} = \left(\frac{\hbar^2}{2}\right) \sum_{a=1}^3 \frac{J_a^2}{J_a}, \quad (2.14)$$

where $J_a = M_a^2$.

As usual, defining operators L_\pm by

$$L_\pm = L_1 \pm iL_2,$$

¹⁴ S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **29**, No. 16 (1955).

¹⁵ B. R. Mottelson and S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Skrifter **1**, No. 8 (1959).

¹¹ So long as we consider the core to be type A , we arrive at the same conclusion even if we add the Coriolis interaction to (2.12).

¹² C. J. Gallagher, Jr., and J. O. Rasmussen, Phys. Rev. **112**, 1730 (1958).

¹³ F. S. Stephens, Jr., F. Asaro, and I. Perlman, Phys. Rev. **100**, 1543 (1955).

we can express the rotational Hamiltonian in the form

$$H_{\text{rot}} = \frac{\hbar^2}{2} \left\{ \left(\frac{1}{4J_1} + \frac{1}{4J_2} \right) (L_+ L_- + L_- L_+) + \frac{1}{J_3} L_3^2 + \left(\frac{1}{4J_1} - \frac{1}{4J_2} \right) (L_+^2 + L_-^2) \right\}. \quad (2.15)$$

Because the operator L_{\pm} shifts the value of K in one unit, the shift of $|\Delta K| = 2$ results from the last term whenever $J_1 \neq J_2$. Thus, in an asymmetric rotator, K is not a good quantum number.

Let us finally write down, then, the wave function which will be used in the following sections.

Noting that the asymmetrical effect of the rotational motion mixes K and $K \pm 2$, the total wave function (2.4) of the system becomes

$$\Psi_{IM}^+(y) = \sum_{\lambda} \mathcal{D}_{M0}^{I*} \Psi_{I0,\lambda}^+ + \frac{1}{\sqrt{2}} \sum_{\lambda} \sum_{K=\text{even} \geq 2} (\mathcal{D}_{MK}^{I*} + (-)^I \mathcal{D}_{M-K}^{I*}) \times \Psi_{IK,\lambda}^+ \quad (2.16)$$

for the levels with positive parity and

$$\Psi_{IM}^-(y) = \sum_{\lambda} \mathcal{D}_{M0}^{I*} \Psi_{I0,\lambda}^- + \frac{1}{\sqrt{2}} \sum_{\lambda} \sum_{K=\text{even} \geq 2} (\mathcal{D}_{MK}^{I*} - (-)^I \mathcal{D}_{M-K}^{I*}) \times \Psi_{IK,\lambda}^- \quad (2.17)$$

for the levels with negative parity. Here λ means some quantum number other than I , K , and l . In (2.16) and (2.17),

$$\Psi_{I0,\lambda}^{\pm} = \sum_l \sum_{L=\text{even}} (lL00 | lLI0) \chi_{l0} \Phi_{L0,\lambda}^I, \quad (2.18)$$

where $I = \text{even}$, $l = \text{even}$ for $K = 0^+$; and $I = \text{odd}$, $l = \text{odd}$ for $K = 0^-$. Also in (2.16) and (2.17)

$$\Psi_{I2,\lambda}^{\pm} = \sum_l \sum_{L=\text{even}} (lL02 | lLI2) \chi_{l0} \Psi_{L2,\lambda}^{I(\pm)}, \text{ etc.}, \quad (2.19)$$

where $l = \text{even}$ for $K = 2$ and $l = \text{odd}$ for $K = 2^-$. The quantum number I may assume any value equal to or greater than 2.

Except for the fact that we do not take $\Psi_{I0,\lambda}^+$ to be equal to $\Psi_{I2,\lambda}^+$, $\Psi_{IM}^+(y)$ as given by (2.16) takes the same form as the corresponding wave function due to Davidov and Filippov.⁸ Wave functions $\Psi_{IM}^{\pm}(y)$ as given by Eqs. (2.16) and (2.17) take the same form as the wave function due to Bohr [Eq. (91) of reference 1], if we take, as we discussed in the present paper, $j = \text{even}$ (odd) for the levels with positive (negative) parity in Bohr's wave function.

At this point, the author would stress that in the following sections any further assumption on the nature

of H_0 and M_a and, hence, of $\Psi_{IK,\lambda}^{\pm}$ will not be introduced. This is the difference between the previous works and the present one.

To calculate the rotational energy, we have to make use of the relations

$$L^{\pm} \mathcal{D}_{MK}^{I*} = \lambda_{IK}^{\mp} \mathcal{D}_{MK \mp 1}^{I*}, \quad (2.20)$$

$$L_3 \mathcal{D}_{MK}^{I*} = K \mathcal{D}_{MK}^{I*},$$

where

$$L^{\pm} = -ie^{\mp i\psi} \left(-\frac{1}{\sin\theta} \frac{\partial}{\partial\varphi} + \cot\theta \frac{\partial}{\partial\psi} \pm i \frac{\partial}{\partial\theta} \right), \quad (2.21)$$

$$L_3 = -i\partial/\partial\psi,$$

and

$$\lambda_{IK}^{\pm} = [(I \mp K)(I \pm K + 1)]^{1/2}. \quad (2.22)$$

III. ENERGY LEVELS OF THE SYMMETRIC ROTATOR

In this section we shall consider the bands $K = 0^{\pm}$, assuming the coupling to the bands $K = 2^{\pm}$ to be small. However, the band $K = 0^+$ may, in some cases, be coupled to another band with $K = 0^+$.

When the rotational energy of the total system is treated as a perturbation of the binding energy of the lowest state of each band, the first-order energy of the total system is given by

$$E^I = E_{\alpha}^{I-I_0} + (\hbar^2/2)(1/J)_{I=I_0,\alpha} I(I+1). \quad (3.1)$$

Here $I_0 = 0$ for $K = 0^+$ and $I_0 = 1$ for $K = 0^-$; α stands for quantum numbers other than I , and $(1/J)_{I=I_0,\alpha}$ is the expectation value of the operator $1/J_1$ ($= 1/J_2$) with respect to the state $\Psi_{I_0,\alpha}^+$. In this approximation, levels belonging to the same rotational band will have the same expectation values of the operator $(1/J_1)$.

In practice it is often necessary to go beyond first-order perturbation theory.^{16,17} When only one excited band, whose lowest energy is given by ϵ , is strongly coupled to the band to which the ground state belongs, the solution of the secular equation derived from the Schrödinger equation (2.12) with (2.15) and (2.16):

$$\begin{vmatrix} E - AI(I+1) & -CI(I+1) \\ -CI(I+1) & E - \epsilon - BI(I+1) \end{vmatrix} = 0.$$

Namely, the following formula gives a more accurate value of the energy:

$$E^{(\pm)} = \frac{1}{2} \{ \epsilon + (A+B)I(I+1) \pm [(\epsilon + (B-A)I(I+1))^2 + 4C^2 I^2 (I+1)^2]^{1/2} \}, \quad (3.2)$$

where

$$A = \frac{1}{2} \hbar^2 (1/J)_{\alpha=0}, \quad B = \frac{1}{2} \hbar^2 (1/J)_{\alpha=1},$$

and

$$C = \frac{1}{2} \hbar^2 (1/J)_{\alpha=0,\alpha'=1}.$$

¹⁶ K. R. Jacob, J. W. Mihelich, B. Harmatz, and T. H. Handley, Phys. Rev. **117**, 1102 (1960).

¹⁷ F. S. Stephens, Jr., R. M. Diamond, and I. Perlman, Phys. Rev. Letters **3**, 435 (1959).

TABLE IV. Application of formula (3.2) to the case of Pu²³⁸. Given the first four values in the second column, and putting values $A=7.37$, $B=11.14$, and $4C^2=13.4$ into Eq. (3.2), we calculate E_0 to be 936 KeV. The last column shows the prediction from the Bohr-Mottelson formula^a: $E=(\hbar^2/2J)I(I+1)-\frac{3}{2}(1/\hbar\omega_\beta)^2 \times (\hbar/J)^2 I^2(I+1)^2$, taking $\hbar^2/2J=7.374$ and $\hbar\omega_\beta=1.10 \times 10^8$.

E_I	Experimental value (keV)	Present calculation (keV)	B.M.
E_2	44.11	44.10	44.11
E_4	146.0	146.1	146.0
E_6	303.7	304.2	303.3
E_8	514	518.4	521.7
E_0	935	936	1100

^a See reference 2.

$\alpha=0$ and 1 mean the band including the ground state and the first excited band with $K=0^+$, respectively.

The wave functions referred to the frame fixed to the core are given by

$$\Psi_I^{(\pm)} = C_a^{I(\pm)} \Psi_{I,\alpha=0} + C_b^{I(\pm)} \Psi_{I,\alpha=1},$$

where

$$C_a^{I(\pm)} = \lambda_I \{ \lambda_I^2 + [1 \pm (1 + \lambda_I^2)^{1/2}] \}^{1/2},$$

$$C_b^{I(+)} = C_a^{I(-)}, \quad C_b^{I(-)} = -C_a^{I(+)} \quad (3.3)$$

$$\lambda_I = \frac{2I(I+1)C}{\epsilon + (B-A)I(I+1)}.$$

The plus and minus signs in (3.2) and (3.3) refer to the upper and lower bands, respectively.

Let us now give a few examples for the symmetric rotator.

(1). $K=0^+$

Given the experimental values¹⁸ of the levels $2^+(44.11)$, $4^+(146.0)$, $6^+(303.7)$, and $8^+(514)$ for Pu²³⁸, we can predict the lowest energy of the next higher band with $K=0^+$ and compare it with the experimental value (935). (Throughout the paper, the unit of energy is taken as 1 keV.) Putting the values $A=7.37$, $B=11.14$, and $4C^2=13.4$ into Eq. (3.2) we get the results shown in Table IV. Note that the deviation from the Bohr-Mottelson formula is appreciable.¹⁹ The same thing was reported by Marklund, Noiien, and Grabowski.²⁰

Table V shows the values of mixing parameters for each state of the lower band. From this table, we see that in each case the mixing of the upper band with the lower band is very small.

¹⁸ I. Perlman and J. O. Rasmussen, in *Encyclopedia of Physics*, edited by S. Flügge (Verlag Julius Springer, Berlin, 1957), Vol. 42, p. 109.

¹⁹ After completion of the paper, Dr. H. Verheul [thesis, Vrije University at Amsterdam, 1962 (unpublished)] informed the author that the formula (3.2) gives the best result compared with other models for Hf¹⁷⁸:

	E_2	E_4	E_6	E_8	E_0	E_2
Exp.	93.2	306.9	632.7	1059.7	1197	1277
(3.2)	93.4	309.3	638.6	1062.1	1197.0	1275.5

²⁰ I. Marklund, B. Van Nooijen, and Z. Grabowski, Nucl. Phys. 15, 533 (1960).

(2). $K=0^-$

A typical example is Ra²²⁶, for which the K value is known to be 0 from the branching ratio of the γ transition.¹³ Experimental and theoretical energy spacing ratios are

$$(E_5 - E_1)/(E_3 - E_1) = 2.85 \quad (\text{experimental value}^{21}),$$

$$= 2.8 \quad [\text{theoretical value from (3.1)}].$$

We should like to stress that one need not consider the octupole vibration model to get the above ratio.

IV. THE ASYMMETRIC ROTATOR

A. Energy-Level Formula

From a generalization of Eq. (3.1), the first-order energy of Eq. (2.12) is given by

$$E_{K^I} = \epsilon_K^{I=I_0} + \frac{1}{2} \hbar^2 \left\{ \frac{1}{\mathcal{J}_K} [I(I+1) - K^2] + \frac{1}{\mathcal{J}_{3K}} K^2 \right\}. \quad (4.1)$$

\mathcal{J}_K and \mathcal{J}_{3K} are the diagonal matrix element of the operators $\frac{1}{2}(1/J_1 + 1/J_2)$ and $1/J_3$, respectively, with respect to the state K :

$$\mathcal{J}_K = 2 \left[\left(\frac{1}{J_1} + \frac{1}{J_2} \right)_K \right]^{-1}, \quad \mathcal{J}_{3K} = \left(\frac{1}{J_3} \right)_K^{-1}. \quad (4.2)$$

Putting the wave function (2.16) or (2.17) into (2.12) and using (2.15) and (2.20), we get the secular equation

$$\begin{vmatrix} E - E_{K^I} & -\mu_{IK} g_{K,K+2} \\ -\mu_{IK+2} g_{K+2,K} & E - E_{K+2^I} \end{vmatrix} = 0, \quad (4.3)$$

for $I=2^+$, $K=0^+$; $I=3^-$, $K=0^-$; $I=4^-$, $K=2^-$. Here, $\mu_{IK} = \mu_{IK+2}$. They are given by: (1). $\mu_{2^+,0^+} = \sqrt{3}/2$; (2). $\mu_{3^-,0^-} = (15)^{1/2}/2$; and (3). $\mu_{4^-,2^-} = (35)^{1/2}/2$. The factor $g_{K,K'}$ in (4.3) is the nondiagonal matrix element of the operator $\frac{1}{2} \hbar^2 (1/J_1 - 1/J_2)$ between the states K and K' . The solution of Eq. (4.3) is given by

$$\begin{Bmatrix} E_{I,K+2} \\ E_{I,K} \end{Bmatrix} = \frac{1}{2} \{ (E_{K+2^I} + E_{K^I}) \pm [(E_{K+2^I} - E_{K^I})^2 + 4\mu_{IK}^2 |g_{K,K+2}|^2]^{1/2} \}. \quad (4.4)$$

TABLE V. Mixing parameters for the $K=0$ band of Pu²³⁸, calculated from formula (3.3).

I	$C_a^{(+)}$	$C_a^{(-)}$
0	0	1.000
2	0.0115	1.000
4	0.0361	0.999
6	0.0698	0.998
8	0.1073	0.994

²¹ F. S. Stephens Jr., F. Asaro, and I. Perlman, quoted in reference 7.

TABLE VI. Intrinsic parameters for nuclei calculated from Eqs. (4.1)–(4.5); a_0^+ , the inverse of the moment of inertia of the $K=0^+$ band: $(\hbar^2/2)(1/\mathcal{J}_0^+)$; a_2^+ , the inverse of the moment of inertia of the $K=2^+$ band: $(\hbar^2/2)(1/\mathcal{J}_2^+)$; b , $\epsilon_2^+ + 2\hbar^2(-1/\mathcal{J}_2^+ + 1/\mathcal{J}_{3,2}^+)$; C , the asymmetric effect, $C=4|g_{2^+,0^+}|^2$. (Davidov and Filippov assume that $\epsilon_2^+=0$. In the present paper, this is not assumed.) These intrinsic parameters provide good agreement for the states $I=4^+$ ($K=2^+$) and $I=6^+$ ($K=0^+$), except for Pu^{238} . On the other hand, Table IV shows the successful prediction for the energy of the state $I=0^+$ ($K=0^+$) of Pu^{238} . Hence, in Pu^{238} , the ground state $K=0^+$ band is coupled more strongly with the first excited $K=0^+$ band. The parameters in the present table are calculated without such a coupling. Accordingly, the parameters for Pu^{238} in this table are not reliable, and are only included for the sake of reference.

Nucleus	a_0^+	a_2^+	b	$10^{-4}C$	Reference
Sm^{152}	20.93	23.53	942.1	1.923	22
Gd^{154}	21.12	21.84	863.5	1.727	23
Dy^{160}	15.25	14.92	870.0	2.226	24
Er^{166}	13.46	12.87	709.1	0.1067	16
Er^{168}	13.33	12.90	744.8	0.07865	16
W^{182}	16.70	17.42	1117.2	0.05865	12
W^{184}	18.64	17.86	796.2	0.2533	25
Os^{190}	32.48	33.40	344.7	1.718	26, 27
Pu^{238}	8.775	8.935	967.9	4.530	18

Here we should take the positive (negative) sign for $E_{I,K+2}$ and the negative (positive) sign for $E_{I,K}$, if $E_{K+2}^I > E_K^I (E_{K+2}^I < E_K^I)$. (4). For the levels of $I=4^+$, in which $K=0^+$ is predominant, the mixing of $K=4^+$ may be neglected, because $K=0^+$ is indirectly coupled with $K=4^+$. Thus, we may use formula (4.4) for $E_{4^+,0^+}$, and with less accuracy, for $E_{4^+,2^+}$. Here $\mu_{4^+,0^+} = \frac{3}{2}\sqrt{5}$. (5). Under the same conditions as for $E_{4^+,0^+}$, the formula (4.4) may be applied to $E_{6^+,0^+}$ with $\mu_{6^+,0^+} = (210)^{1/2}/2$. Finally, in the case of (6). $E_{1^-,0^-}$, (7). $E_{2^-,2^-}$, and (8). $E_{3^+,2^+}$, we may use the formula

$$E_{I,K} = E_K^I, \quad (4.5)$$

because there are no coupled levels.

If we assume

$$\mathcal{J}_0^+ = \mathcal{J}_2^+, \quad (4.6)$$

we have the relation

$$E_{3^+,2^+} = E_{2^+,0^+} + E_{2^+,2^+}. \quad (4.7)$$

This is the relation put forth by Davidov and Filippov.³ As we saw in Sec. II, the intrinsic wave function of the band $K=0^+$ will not be identical with that of the band $K=2^+$. However, the experimental results as summarized by Jacob *et al.*¹⁶ (see their Table XII) show that this relation is nevertheless fairly well satisfied. This fact may indicate that the number of extra particles in the sense of the present paper is rather small.

B. Application to Levels with Positive Parity

Using the energies of the first two excited states of the band $K=0^+$ involving the ground state and those of the band $K=2^+$, we can get values for a few intrinsic quantities as shown in Table VI.^{12,16,18,22–27} We can also

²² O. Nathan and S. Hultberg, Nucl. Phys. **10**, 118 (1958).

²³ K. S. Toth and J. O. Rasmussen, Phys. Rev. **115**, 150 (1959).

²⁴ O. Nathan, Nucl. Phys. **4**, 125 (1957).

TABLE VII. Calculated energy value of the state $I=4^+$, $K=2^+$ and comparison with experimental values. The parameters used are listed in Table VI. References are the same as those of Table VI.

Nucleus	Experimental value (keV)	Present calculation (keV)	Exp/calc
Sm^{152}	...	1464	...
Gd^{154}	1399.7(?)	1352	1.035
Dy^{160}	...	1240	...
Er^{166}	957.2	970.7	0.986
Er^{168}	996.3	989.0	1.007
W^{182}	1500.6	1468.0	1.022
Os^{190}	956	1118.7	0.855
Pu^{238}	...	1268.3	...

predict the energies of more highly excited states. Thereby, the assumption (4.6) will *not* be used.

Using the values in Table VI, we have calculated the energies of the state $I=4^+$ of the band $K=2^+$, and $I=6^+$ of the band $K=0^+$. The results are listed in Tables VII and VIII, respectively. In carrying out the calculation, we have assumed the coupling to the state of the band $K=4^+$ to be negligible in both cases.

For the case $I=4^+$, $K=2^+$, we see that the agreement between experiment and theory is quite good: There is less than a 4% difference for all nuclei, with the exception of Os^{190} . For that particular nucleus, the discrepancy may be an indication that the coupling with the state of $K=4^+$ is no longer negligible.

For the state $I=6^+$, $K=0^+$, we also have good results in the case of Er^{166} , Er^{168} , W^{182} , and Os^{190} . The success in Os^{190} is especially interesting in that the deviation from the $I(I+1)$ rule or the $aI(I+1) + bI^2(I+1)^2$ rule is serious, as shown in Table IX, whereas formula (4.3) allows a good fit. Also it is very interesting to note that in Os^{190} , $(E_2^6 - E_0^6)^2 \ll 4\mu_{6^+,0^+}^2$

TABLE VIII. Calculated energy value of the state $I=6^+$, $K=0^+$ and comparison with experimental values. The parameters used are listed in Table VI. We can not get satisfactory results for Pu^{238} using the parameters in Table VI. This means that in Pu^{238} the coupling of the band $K=0^+$ with the band $K=2^+$ is not important. On the other hand, the coupling of this band with the first excited band of $K=0^+$ is important, as Table IV shows.

Nucleus	Experimental value (keV)	Present calculation (keV)	Exp/calc
Sm^{152}	...	677.6	...
Gd^{154}	...	682.6	...
Dy^{160}	...	378.8	...
Er^{166}	545.3	545.5	0.9996
Er^{168}	548.9	546.0	1.005
W^{182}	680.38	694.8	0.9792
W^{184}	...	751.3	...
Os^{190}	1048	1044.7	1.003
Pu^{238}	303.7

²⁵ C. J. Gallagher, Jr., D. Strominger, and J. P. Unik, Phys. Rev. **110**, 725 (1958).

²⁶ O. B. Nielsen, N. O. R. Poulsen, R. K. Shelin, and B. S. Jensen, Nucl. Phys. **10**, 475 (1959).

²⁷ W. R. Kane, G. T. Emery, G. Scharff-Goldhaber, and M. McKeown, Phys. Rev. **119**, 1953 (1960). The author thanks Dr. Emery for a private communication on this matter.

$\times |g_{2^+0^+}|^2$; i.e., the asymmetric effect is of paramount importance determining the energy level.

For Pu^{238} we can not get the correct value of $E_{6^+,0^+}$ by considering the asymmetric effect of the rotational motion. On the other hand, Table IV lists quite satisfactory results for this nucleus. Hence, we conclude that in Pu^{238} the band $K=0^+$ including the ground state is coupled strongly with the first excited band with $K=0^+$ and not with the band $K=2^+$. For this reason we cannot rely on the intrinsic parameters for Pu^{238} as given in Table VI.

At the present time, data are lacking for similar analyses on other nuclei. When such becomes available, we may gain important information on the intrinsic excitations in the excited states of even-even nuclei.¹⁹

C. Application to Levels with Negative Parity

At the present time information on levels with negative parity is lacking. The only exception is W^{182} ; it has three bands which have been assigned negative parity.¹² However, there is a problem in this assignment; the K value of the level sequence 1^- , 3^- , and 5^- has tentatively been set to 1^- from the ordinary intensity rule^{7,28} but the estimate of the gamma-ray intensity is somewhat uncertain even in this nucleus. Furthermore, since three bands are close together and presumably strongly coupled, the intensity rule might not be applicable at all. In view of this uncertainty, we should study the case for both $K=1^-$ and $K=0^-$.

If we assume first that the level sequence 1^- , 3^- , and 5^- belongs to the band $K=0^-$, then the description in Sec. IV A applies. The formulas, though simple, contain many parameters, especially for the levels of negative parity. To reduce the number of parameters, we take as a first approximation;

$$J_0^- \approx J_2^- \approx J_4^-, \quad (4.8)$$

and

$$g_{2^-0^-} \approx g_{4^-2^-}. \quad (4.9)$$

Then we have the relation

$$E_{3^-,0^-} + E_{3^-,2^-} = E_{1^-,0^-} + E_{2^-,2^-} + 16(\hbar^2/2)(1/J_0^-), \quad (4.10)$$

TABLE IX. Energy levels of Os^{190} . In the present calculation, the energies of first two levels in this table and the energies of first two lowest levels of the band $K=2^+$ were taken from experiment.^{a,b} The energy of the level $I=6^+$ is then calculated.

Level	Experimental value	Present calculation	$I(I+1)$ rule	$\frac{aI(I+1)}{+bI^2(I+1)^2}$ rule
2^+	186	186.0	186.0	186.0
4^+	547	546.6	619.4	547
6^+	1048	1045	1302	818.6

^a See reference 26.

^b See reference 27.

²⁸ G. Alaga, K. Adler, A. Bohr, and B. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-Fys. Medd. **29**, No. 9 (1955).

from which we get

$$(\hbar^2/2)(1/J_0^-) = 14.72$$

using the experimental values of $E_{3^-,0^-}$, $E_{3^-,2^-}$, $E_{1^-,0^-}$, and $E_{2^-,2^-}$. Next using the formula for $I=1^-$ and $I=2^-$, we have

$$b_0 \equiv \epsilon_0^{-1} = 1225.7,$$

and

$$b_2 \equiv \epsilon_2^{-2} + 2\hbar^2[-(1/J_2^-) + (1/J_{3,2^-})] = 1201.2.$$

These results show that $E_{0^-3^-} > E_{2^-3^-}$, accordingly $E_{3^-,0^-} > E_{3^-,2^-}$, in agreement with the experimental level ordering.

Then using the formula for $I=3^-$, we get

$$4|g_{2^-,0^-}|^2 = 108.41.$$

Next using the assumptions (4.8) and (4.9), we get from the formula for $I=4^-$ and the experimental energy value of the lower level of $I=4^-$,

$$b_4 \equiv \epsilon_4^{-4} + 8\hbar^2[-(1/J_4^-) + (1/J_{3,4^-})] = 1225.3. \quad (4.11)$$

In deriving this value, it was immaterial whether $E_{4^-,2^-} > E_{4^-,4^-}$ or $E_{4^-,2^-} < E_{4^-,4^-}$. However, since the resulting value of b_4^- is greater than that of b_2^- , it must be that $E_{4^-,4^-} > E_{4^-,2^-}$. This result is the inverse of the present experimental assignment. However, we should bear in mind that, owing to the coupling, the usual intensity rule is no longer applicable to the assignment of the K values in this energy region of W^{182} .

Adding the formula for $I=4^-$, $K=2^-$ to that for $I=4^-$, $K=4^-$ and again using Eq. (4.8), we get the equation

$$E_{4^-,2^-} + E_{4^-,4^-} = b_2^- + b_4^- + 40(\hbar^2/2)(1/J_0^-). \quad (4.12)$$

Finally, we obtain the value of $E_{4^-,4^-} = 1527.4$. The experimental value of the upper level of $I=4^-$ is 1533.6. The ratio of the experimental value to the theoretical one is therefore 1.017, and the fit is very good.

If we next assume that the sequence 1^- , 3^- , and 5^- belongs to the band $K=1^-$, then the wave function of the system is the B_2a -type or the B_3a -type with odd values of L . Since the levels with $K=1^-$ do not couple to the bands with $K=2^-$ or $K=4^-$, and since we have no band with an experimental assignment of $K=3^-$, the levels 1^- , 3^- , and 5^- must form an isolated band. In this case, the energy of these levels will be given simply by the $I(I+1)$ rule. Applying this rule, we are able to select out, as belonging to the $K=1^-$ band, the 1450.9-keV level from the two levels with $I=3^-$, and the 1659.8-keV level from three levels with $I=5^-$. Since the level of $I=3^-$ belonging to the band $K=2^-$ is not coupled to the other bands, we can estimate the value of $(\hbar^2/2)(1/J_2^-)$ and b_2^- from the experimental values of the levels $I=2^-$ and $I=3^-$ of $K=2^-$. The levels with $K=4^-$ obey the formula (4.4). Finally, we obtain the value of b_4^- from the formula (4.11). However, the value of b_4^- thus obtained is too large and it

makes the quantity $4|g_{4-2}|^2$ negative when we determine its value by using the formula (4.4), applied to $I=4^-, K=2^-$. This is not satisfactory.

From the above discussion it would therefore seem reasonable to assume that the K value of the level sequence $1^-, 3^-,$ and 5^- in W^{182} is zero. At the same time, this example justifies the use of the wave function (2.17).

V. THE γ -TRANSITION PROBABILITIES

If we assume only coupling between the states $I=2^+$ of two bands with $K=0^+$, the ratio of the reduced $E1$ -transition probability between the levels $I=1^-, K=0^- \rightarrow I=0^+, K=0^+$ to that between levels $I=1^-, K=0^- \rightarrow I=2^+, K=0^+$ is given by

$$\frac{B(E1; 1^0- \rightarrow 0^+0^+)}{B(E1; 1^0- \rightarrow 2^+0^+)} = \frac{|(1100|1100)|^2}{(C_a^{(-)})^2 |(1100|1120)|^2} = 0.5(C_a^{(-)})^{-2}. \quad (5.1)$$

Here $C_a^{(-)}$ is given by (3.3). In Pu^{238} , this ratio is 0.5, to be compared with the experimental value¹³ of 0.6 ± 0.15 ; $C_a^{(-)}=1$, as shown in Table V. We want to stress again that insofar as the ratio of the level spacing (Sec. III) and the ratio of the $E1$ transitions are concerned, it is not necessary to use the liquid drop picture of the core, nor the octupole vibration model for the band $K=0^-$.

In the event that the coupling between levels $I=2^+, K=0^+$ and $I=2^+, K=2^+$ is rather strong, $C_a^{(-)}$ in the above formula should be replaced by $a_{20}^{(-)}$, defined by

$$a_{IK}^{(\pm)} = \lambda_{IK} \{ \lambda_{IK}^2 + [1 \pm (1 + \lambda_{IK}^2)^{1/2}] \}^{-1/2},$$

$$\lambda_{IK} = \frac{2\mu_{IK} g_{K+2, K}}{E_{K+2}^I - E_K^I}. \quad (5.2)$$

In general, the calculation of other γ -transition probabilities requires an explicit form for the wave function. This is achieved only at the expense of further assumptions. Rather than do this, we shall simply discuss some of the general aspects of the problem.

Predictions using our wave functions will not differ greatly from those of the usual intensity rule, as can be seen from the following: If the mixing of the bands

$K=0^+$ and 2^+ comes from the asymmetric effect, then using (5.2), we can express the wave function (2.16) in the form

$$\Psi_{IM, 0^+} = a_{I0}^{(-)} \mathcal{D}_{M0}^{I*} \varphi_{I0, \lambda} - a_{I0}^{(+)} (1/\sqrt{2}) (\mathcal{D}_{M2}^{I*} + \mathcal{D}_{M-2}^{I*}) \varphi_{I2, \lambda} \quad \text{for } K=0^+,$$

and

$$\Psi_{IM, 2^+} = a_{I0}^{(+)} \mathcal{D}_{M0}^{I*} \varphi_{I0, \lambda} + a_{I0}^{(-)} (1/\sqrt{2}) (\mathcal{D}_{M2}^{I*} + \mathcal{D}_{M-2}^{I*}) \varphi_{I2, \lambda} \quad \text{for } K=2^+.$$

For example, $a_{20}^{(+)}=0.06234$, $a_{20}^{(-)}=0.9981$, $a_{40}^{(+)}=0.2171$, and $a_{40}^{(-)}=0.9761$ for Sm^{152} ; $a_{20}^{(+)}=0.01636$, $a_{20}^{(-)}=0.9999$, $a_{40}^{(+)}=0.06351$, and $a_{40}^{(-)}=0.9980$ for Er^{168} . These values are to be compared with the following values which are used in deriving the usual intensity rule: $a_{20}^{(+)}=a_{40}^{(+)}=0$, $a_{20}^{(-)}=a_{40}^{(-)}=1$. One would, therefore, expect the theoretical branching ratio using the wave function (5.3) to agree with experiment whenever the usual intensity rule holds. This rule should hold in those regions where mixing due to the asymmetric effect is very small, namely at low-lying levels in the rare-earth region (as seen from the column 5 of Table VI).

VI. CONCLUSIONS

It has been shown that the present approach gives some information on intrinsic excitation of even-even nuclei. The validity of this paper does not depend on the exact nature of the moment-of-inertia operator M_a .

Also, it has been shown that the rotational levels of negative parity can be treated on the same basis as those of positive parity in every respect: with regard to the rotational property of the wave function, the energy levels, and the γ -transition probabilities.

The wave function employed in the present paper gives very good results with respect to energy levels and with respect to those γ -transition probabilities where the usual intensity rule holds.

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APPENDIX

The Explicit Form of the Wave Function $\Psi_{IM}^{(3)}$ in (2.4).

Introducing the wave function $\chi_{I\Omega}^{(\pm)}$ for $\Omega \neq 0$,

$$\chi_{I\Omega}^{(\pm)} = (1/\sqrt{2})(\chi_{I\Omega} \pm \chi_{I-\Omega}), \quad (A1)$$

$\Psi_{IM}^{(3)}$ in (2.4) can be expressed as

$$\Psi_{IM}^{(3)} = \Psi_{IM}^{(3a)} + \Psi_{IM}^{(3b)} + \Psi_{IM}^{(3c)}, \quad (A2)$$

where

$$\Psi_{IM}^{(3a)} = \sum_{LI} \sum_{\substack{\Omega > 0 \\ (K=0)}} \frac{1}{2} (lL\Omega - \Omega | lLI0) \mathcal{D}_{M0}^{I*} \{ (\chi_{I\Omega}^{(+)} \Phi_{L\Omega}^{I(+)} - \chi_{I\Omega}^{(-)} \Phi_{L\Omega}^{I(-)}) [1 + (-)^{l+L-I}] \\ + (\chi_{I\Omega}^{(-)} \Phi_{L\Omega}^{I(+)} - \chi_{I\Omega}^{(+)} \Phi_{L\Omega}^{I(-)}) [1 - (-)^{l+L-I}] \},$$

$$\Psi_{IM}^{(3b)} = \sum_{LI} \sum_{\substack{K=\Omega \\ (K>0, \Omega>0)}} (ILK0|LIK) (\mathcal{D}_{MK}^{I(+)*} \chi_{IK}^{(+)} \Phi_{L0}^I + \mathcal{D}_{MK}^{I(-)*} \chi_{IK}^{(-)} \Phi_{L0}^I),$$

and

$$\begin{aligned} \Psi_{IM}^{(3c)} = \sum_{LI} & \left(\sum_{\substack{K-\Omega>0 \\ (K>0, \Omega>0, K \neq \Omega)}} + \sum_{\substack{K-\Omega<0 \\ (K>0, \Omega>0, K \neq \Omega)}} \right) \\ & \times \left\{ \frac{1}{\sqrt{2}} (IL\Omega K - \Omega |LIK) [\mathcal{D}_{MK}^{I(+)*} (\chi_{I\Omega}^{(+)} \Phi_{L, K-\Omega}^{I(+)} + \chi_{I\Omega}^{(-)} \Phi_{L, K-\Omega}^{I(-)}) \right. \\ & \quad \left. + \mathcal{D}_{MK}^{I(-)*} (\chi_{I\Omega}^{(+)} \Phi_{L, K-\Omega}^{I(-)} + \chi_{I\Omega}^{(-)} \Phi_{L, K-\Omega}^{I(+)}) \right] \\ & + \frac{1}{\sqrt{2}} (IL\Omega K + \Omega |LIK) [\mathcal{D}_{MK}^{I(+)*} (\chi_{I\Omega}^{(+)} \Phi_{L, K+\Omega}^{I(+)} - \chi_{I\Omega}^{(-)} \Phi_{L, K+\Omega}^{I(-)}) \\ & \quad \left. + \mathcal{D}_{MK}^{I(-)*} (\chi_{I\Omega}^{(+)} \Phi_{L, K+\Omega}^{I(-)} - \chi_{I\Omega}^{(-)} \Phi_{L, K+\Omega}^{I(+)}) \right] \Big\}. \end{aligned}$$

Corresponding to (2.8), we have

$$C_2^{\tau} \chi_{I\Omega}^{(\pm)} = \pm (-)^{-L} \chi_{I\Omega}^{(\pm)}, \quad C_2^{\nu} \chi_{I\Omega}^{(\pm)} = \pm (-)^{-(I+\Omega)} \chi_{I\Omega}^{(\pm)}, \quad \text{and} \quad C_2^z \chi_{I\Omega}^{(\pm)} = (-)^{-\Omega} \chi_{I\Omega}^{(\pm)}, \quad (A3)$$

for $\Omega \neq 0$.

Proton Scattering by Ni⁶⁴ and Zn⁶⁴ at 9.6 and 11.7 MeV*

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Differential cross sections for elastic scattering of protons from Ni⁶⁴ and Zn⁶⁴ were measured and compared. The observed shift in the positions of the maxima and minima of the differential cross sections is shown to be consistent with the presence of a symmetry energy term $[C(N-Z)/A]$ in the real nuclear potential of the optical model. It is found that $C \approx 40$ MeV, in reasonable agreement with the results of other observations.

Comparison of the back-angle data yields an estimate of about 15 mb for the compound elastic-scattering cross section of Zn⁶⁴ at 9.60 MeV. The 11.7-MeV data show no contribution from compound elastic scattering.

I. INTRODUCTION

ONE assumption adopted in the early development of the optical model^{1,2} of the nucleus is that the depth of the real potential is independent of the mass number. This seemed quite reasonable in the beginning, in that any restrictions which could be placed on the many parameters used in fitting experimental data were welcome. This assumption has turned out to be quite fruitful. Remarkable success has been reported in describing the results of proton interactions with a wide range of nuclei over a large breadth of energies.

More recently, theoretical studies³⁻²² have shown that

there is reason to expect a difference between the proton-nucleus potential and the neutron-nucleus potential. The clearest contribution to this effect seems to be a

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